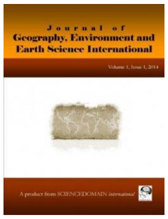

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A Simple Physical Model of River Meandering

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Authors' contributions

This work was carried out in collaboration between all authors. Author VGZ designed the physical basis of the study, managed the literature searches, and wrote the first draft of the manuscript. Author OAG fulfilled all analytical and numerical calculations and designed the program for them. All authors read and approved the final manuscript.

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ABSTRACT

Aims: Rivers often are twisting (meandering). There is no a simple physical model, which would explain the cause of formation of meanders and would describe their main (general) features, abstracting from the peculiarities of the real rivers. The resent work is devoted to creation and discussion of such model.
Study Design: We describe general features of river meandering in the framework of a simple physical model based on the law of constancy of the total stream velocity and action of gravity.
Place and Duration of Study: Institute of Materials, Khabarovsk, Russia; Institute of Applied Mathematics, Khabarovsk, Russia; 2013-2014.
Methodology: We consider a water stream flowing with a constant average velocity along a valley having slopes of constant inclination.
Results: We have found that the stream deviations at different obstacles can play a role of the reason of meandering. The sinousity of a stream depends on the ratio of the slope and valley steep angles; and its mean value is about 1.5 in accordance with observed geography data.

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Conclusion: General features of river meandering may be understood in the framework of a simple physical model based on influence of gravity and the law of constancy of the total stream velocity. We have found that the stream deviations at different obstacles can play a main causal role for meandering. The sinousity *S* of a stream depends on the ratio of the slope and valley steep angles β/α and is equal to 1.507 for β/α=1 and the deviation angle δ=90º. For other cases the value of *S* lies between 1 and 2 for δ≤90º and has a maximal limit of 2.4 for δ≤90º at δ=123.4º.

Keywords: Meandering; sinousity; physical model.

1. INTRODUCTION

Rivers are not straight; their current is twisting (meandering). The ratio of the curved length *L* of the river to length of a straight path *P* is called as sinuosity *S* (*S*=*L*/*P*). In the nature, meandering of the river depends on many aspects of a surrounding relief, features of the breeds forming its valley and many others. As a result, observed sinuosity of the rivers varies in the wide range of sizes: from hardly exceeding unit to several ones. Seeking to explain the nature of meandering the majority of researchers tries to take into account thin hydrodynamic features of a current (turbulence, secondary flows, etc.) [1,2,3,4], interaction of the river with banks (washout, material transfer, etc.) [5], and stochastic deviations of the stream from a straight way [6,7]; the hypothesis of an important role of casual obstacles was stated by Popov [8]. The most researchers reported that the average sinuosity is equal to 1.5-1.6 and rarely overcomes 2.0 [9,10,11,12], but stochastic calculations predict a value of π [6]. It is known that instability of a straight moving is a fundamental feature of a river flow [13,14,15]. However, even now there is no a simple physical model, which would explain the cause of formation of meanders and would describe their main (general) features, abstracting from the peculiarities of the real rivers. The recent work is devoted to creation and discussion of such model.

2. MODEL

River valleys can have different profile. We will consider here only the simplest of them: V-like one, which has slopes of a constant inclination. This case allows obtaining the main formulas analytically. Valleys that are more complicated may be considered by the same way numerically.

Fig. 1 demonstrates the common details of a V-like river valley. The straight direction of its bottom is characterized with a momentum ***A***, which has the angle α in relation to the X axis. The valley slope forms the angle β between it and the Y axis and may be characterized by a ***B*** momentum which is orthogonal to ***A***. Firstly the valley has no channel, thus water can flow straightly in the ***A*** direction under the action of gravity. If the width and the depth of the stream are stable its velocity V_0 is constant and caused by equilibrium of the gravity force and the forces of friction.

Let us consider the movement of water along a valley with the perfect plate slopes. This movement is straight in the absence of obstacles (Fig. 2, the 0-1 region). If the stream meets a hard obstacle in the point 1, it changes its direction and the deviation angle δ will be depending on the angle of collision γ and many other conditions. If, for example, δ = 90º the stream will rise on the slope moving at the same time down on a valley. Its summary movement (the first meander) is shown by the curve 1-2-3. Then the stream goes to the opposite slope and repeats its movement by the same way, after that the whole picture repeats periodically if there are no other obstacles.

In each moment *t* the water situated in the point [*a*(*t*), *b*(*t*)] takes part in two orthogonal movements characterized by $V_A(t)$ and $V_B(t)$ velocities, which submit to the following condition:

$$V_A^2(t) + V_B^2(t) = V_0^2 . \tag{1}$$

We see that only one from the $V_A(t)$ and $V_B(t)$ variables may be called independent. Let $V_A(t)$ be an independent variable. Then we can write simple Newton's equations for movement of water stream:

$$\frac{dV_A}{dt} = g \sin\alpha , \tag{2}$$

$$\frac{dV_B}{dt} = -g\sin\beta + \frac{\partial V_B}{\partial V_A}\frac{\partial V_A}{\partial t} = -g\sin\beta - Z^{(\pm)}\frac{\sqrt{V_0^2 - V_B^2}}{V_B}g\sin\alpha\ , \quad (3)$$

where $Z^{(\pm)}$ is +1 or -1 and the choice is carried out in view of continuity of a stream, velocities and their derivatives.

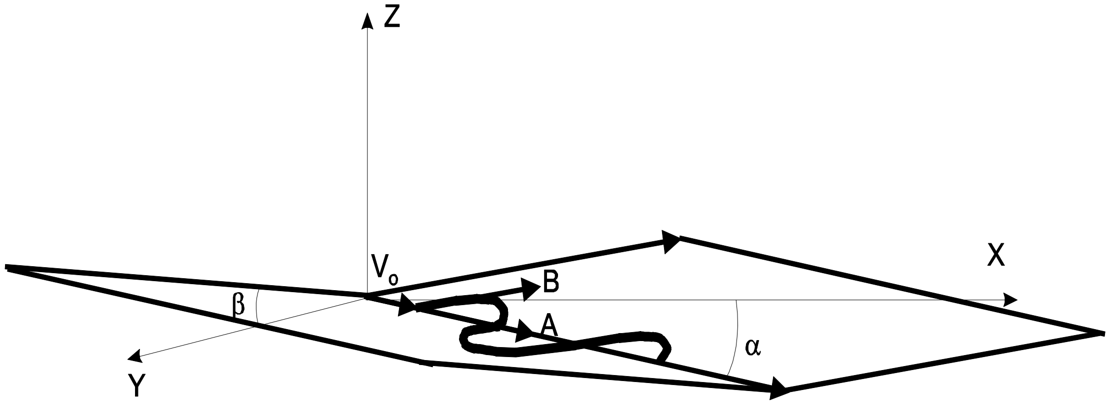

Fig. 1. A scheme of a perfect river valley

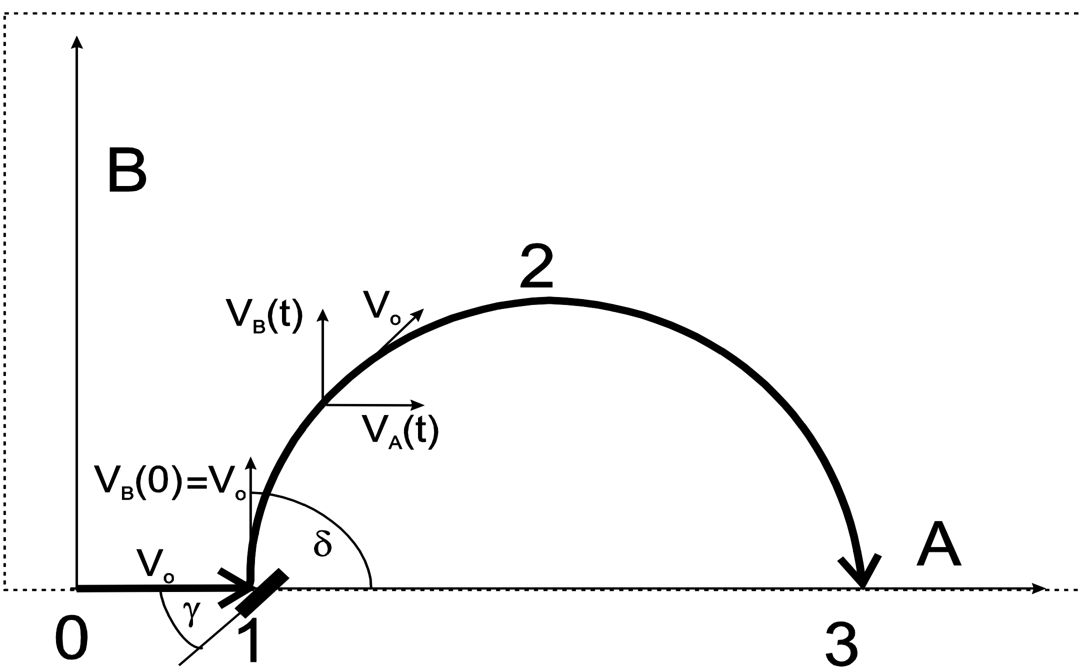

Fig. 2. A scheme of formation of a meander loop: γ = 45º and δ = 90º

Using (3) we can calculate the connection between V_B and t:

$$\int_{V_B(0)}^{V_B} \frac{V_B dV_B}{V_B g\sin\beta + Z(\pm)\sqrt{V_0^2 - V_B^2}\,g\sin\alpha} = t\ , \quad (4)$$

where $V_B(0)=V_0\sin\delta$ is the initial V_B velocity at t=0.
Integrating in (4) we obtain

$$t = J(V_B) - J(V_B(0)), \tag{5}$$

$$J(V_B) = \frac{V_B \sin\beta - \sin\alpha \cdot Z^{(\pm)}\sqrt{V_0^2 - V_B^2}}{g\left(\sin^2\alpha + \sin^2\beta\right)} + \frac{V_0 \sin\alpha \sin\beta}{g\left(\sin^2\alpha + \sin^2\beta\right)^{3/2}} \ln\left|\frac{\sqrt{\sin^2\alpha + \sin^2\beta}\cdot Z^{(\pm)}\sqrt{V_0^2 - V_B^2} + V_0\sin\beta}{\sqrt{\sin^2\alpha + \sin^2\beta} + V_0 \sin\alpha}\right|. \tag{6}$$

$$J(V_B(0)) = J(V_B = V_B(0)).$$

Inversing numerically the $t(V_B)$ dependence we receive values of $V_B(t)$ and then calculate $V_A(t)$:

$$V_A(t) = Z^{(\pm)}\sqrt{V_0^2 - V_B^2(t)}, \qquad V_A(0)=V_0\cos\delta. \tag{7}$$

The expressions (5), (6) and (7) allow us to find numerically the paths down the valley $A(t)$ and up the slope $B(t)$:

$$A(t) = \int_0^t V_A(t)dt, \tag{8}$$

$$B(t) = \int_0^t V_B(t)dt. \tag{9}$$

Because *A(t)* is the path down the valley, *B(A)* describes the shape of the stream channel.

For the sinuosity S we have obtain an analytic expression (for δ=90º):

$$S = 2\sqrt{1+\frac{\sin^2\beta}{\sin^2\alpha}}\;\frac{\left(1+\frac{\sin\beta}{\sin\alpha}\right)\sqrt{1+\frac{\sin^2\beta}{\sin^2\alpha}} + \frac{\sin\beta}{\sin\alpha}\cdot\ln\left|\dfrac{\frac{\sin\beta}{\sin\alpha}}{\left(1+\sqrt{1+\frac{\sin^2\beta}{\sin^2\alpha}}\right)\left(\frac{\sin\beta}{\sin\alpha}+\sqrt{1+\frac{\sin^2\beta}{\sin^2\alpha}}\right)}\right|}{1+\frac{\sin^2\beta}{\sin^2\alpha}+\frac{\pi}{2}\frac{\sin\beta}{\sin\alpha}\left(\frac{\sin^2\beta}{\sin^2\alpha}-1\right)-2\frac{\sin^2\beta}{\sin^2\alpha}\cdot\ln\left|\frac{\sin\beta}{\sin\alpha}\right|}. \tag{10}$$

3. RESULTS

Our equations allow us to image the shape of meanders and to calculate the value of sinuosity if we know the average water velocity V_0, the deviation angle δ and parameters of the valley α and β. The picture is periodical if there is only one obstacle. In nature, there are many obstacles; however, each of them leads to the same problem as described above, thus we consider below only cases with one obstacle.

3.1 Deviation Forward-aside: δ≤90º

Using typical for plain rivers values V_0=2 m/s, α =0.0005 radian and the well-known value g=9.81 m/s^2, we have plotted $B(A)$ for δ≤90º for β=α (Fig. 3).

We see that for small deviations (δ = 20º, 40º) the shape of meanders looks like sin. However, it is impossible to describe meanders of real rivers with large deviations of stream by sin functions (for example, for δ = 90º), but our model describes them easily.

In all cases V_B is equal to zero and $V_A=V_0$ in the top point of a curve. In the case of δ=90º and α=β the sinuosity $S = 4 \cdot \left(1 - \frac{\ln(\sqrt{2}+1)}{\sqrt{2}}\right)$ =1.507 that is a little smaller than π/2=1.570...

The value S depends on the ratio β/α and aspires to 2.0 with β/α→0 (Fig. 4, the left panel). Decreasing of δ leads to decreasing of the sinousity (Fig. 4, the right panel).

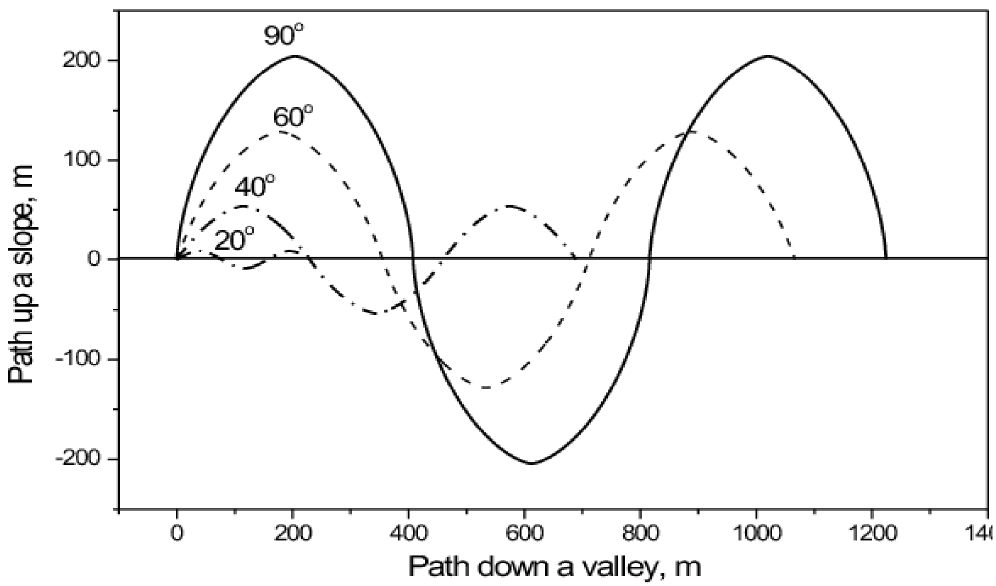

Fig. 3. Three-loops meander curves $B(A)$ for δ≤90º, α=β=0.0005 radian, V_0=2 m/s

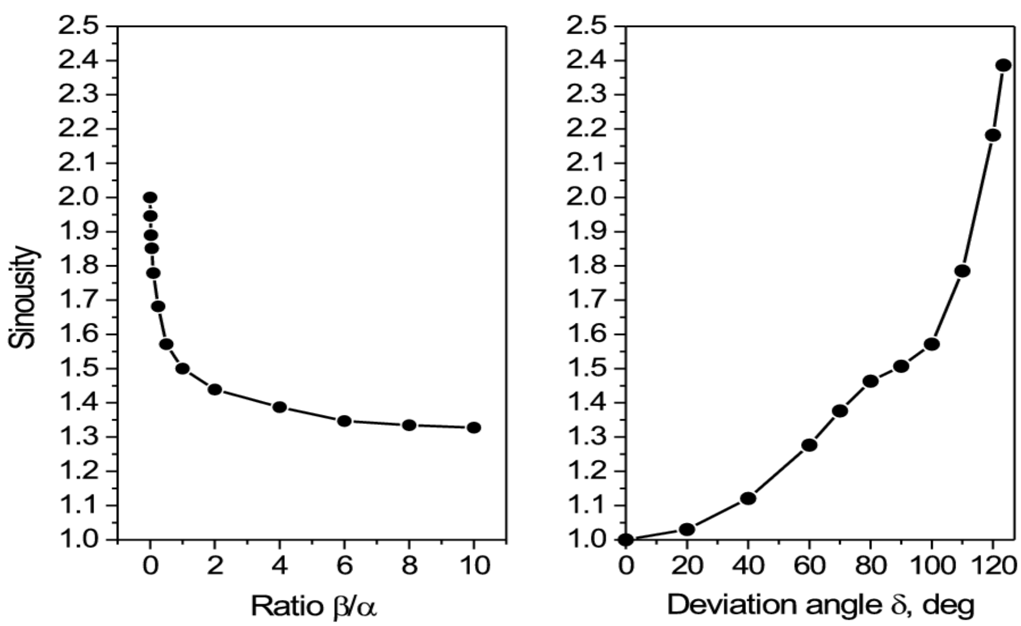

Fig. 4. Dependence of sinousity: left) on the ratio β/α (δ=90º); right) on the deviation angle δ (β/α=1)

The amplitude of a meander also aspires to some limited value, and this value is equal to $\frac{V_0^2}{g\sin\alpha}\cdot\frac{\pi}{4}$ (for δ=90º and β/α=1). It must be marked that the case β=0º has no connection to meandering; it describes a river movement down a wide plain valley and results in creation of river arms (distributaries) with a straight flowing in the ***A*** direction after t=t_{max} (see Fig. 5). This case requires a special investigation out of the borders of the present work. The case of β/α>1 describes valleys with slopes steeper than the valley falling. We suppose that our simple model is not correct for β/α»1 (especially for β→90º), thus in Fig. 4 we have limited by β/α=10.

3.2 Deviation Back-aside: δ>90º

The behavior of the water stream with intial deviations δ>90º is shown in Fig. 6. One can see that the sinousity increses as δ growths. However, there is a geometry restriction for meandering with large initial deviations. This restriction is caused by a contact of the nearest loops and leads to the maximal value of δ_{max}=123.4º and S_{max}≈2.4. For comparison, Da Silva [4] reported δ_{max}=126º and S_{max}≈8.5 for the sin-generated meander curves. We suppose that our results are more correct because they follow from a realistic physical model.

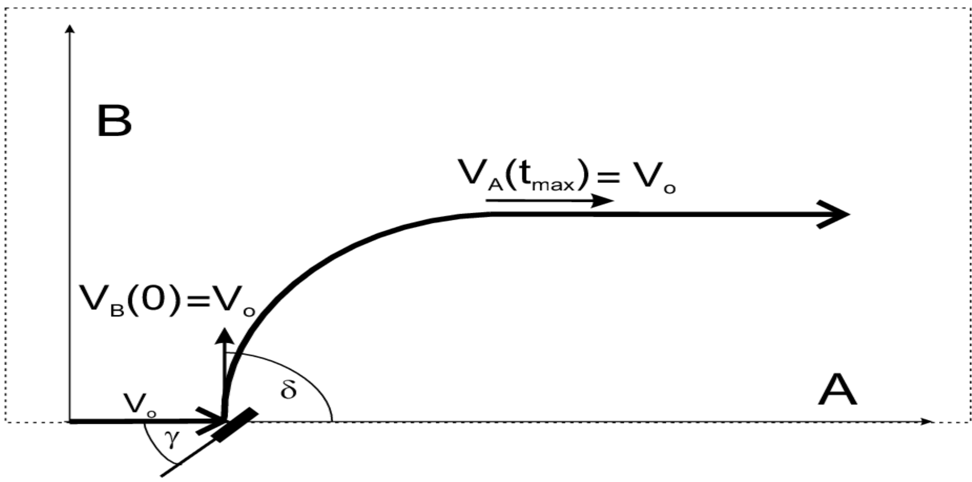

Fig. 5. A scheme of the stream movement in the case of β=0º, δ=90º

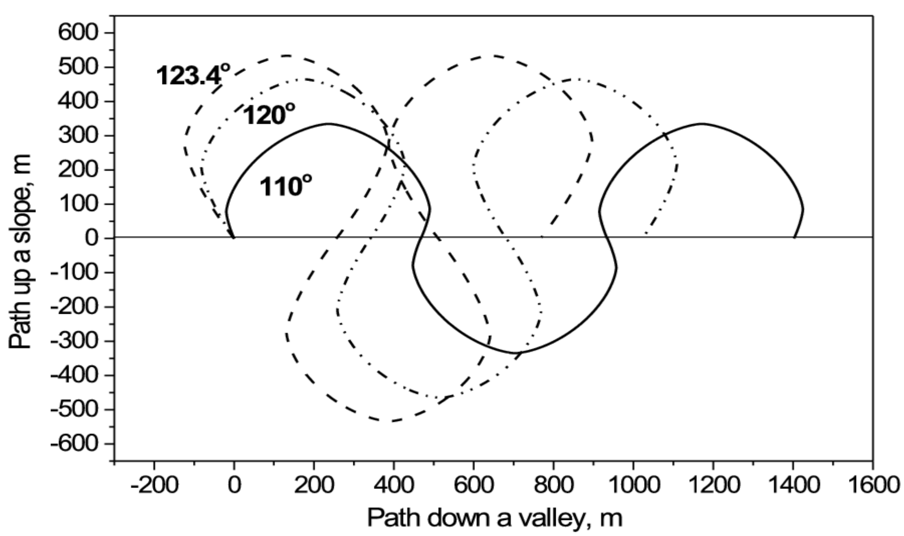

Fig. 6. The curves *B*(*A*) for δ>90º

4. CONCLUSION

Summarizing our results we can conclude that general features of river meandering may be understood in the framework of a simple physical model based on influence of gravity and the law of constancy of the total stream velocity. We have found that the stream deviations at different obstacles can play a main causal role for meandering. The sinousity *S* of a stream depends on the ratio of the slope and valley steep angles β/α and is equal to 1.507 for $\beta/\alpha=1$ and the deviation angle $\delta=90^{o}$. For other cases the value of *S* lies between 1 and 2 for $\delta\leq 90^{o}$ and has a maximal limit of 2.4 for $\delta\leq 90^{o}$ at $\delta=123.4^{o}$. Obviously, meandering of natural rivers is also caused by peculiarities of the surrounding relief and by many other reasons; however we believe that our model will help to understand this phenomenon much better.

Da Silva [4] wrote: "According to Yang, most theories "emphasize some special phenomena observed in meandering channels and neglect the physical reasoning which creates them". From the debates of these theories, eventually the idea settled that if an explanation for why meandering initiates is be generally accepted, it should not fail to explain: 1 – why the wavelength of meanders should be $\Lambda_M \approx 6B$, and 2 – why meanders occur even when there is no sediment transport…" (Here B is the channel width.) Da Silva believes that turbulence is the reason of meandering. However, it is only a qualitative reasoning. He does not offer a physical model, which mathematically would bring a form of meanders out of turbulence. He does not discuss any other physical models. He only specifies, what conditions such models have to meet.

It seems to us that his conditions (1 and 2) are not connected physically and can have different nature. Our model satisfies the second condition: it describes a meandering without sediment transport. And it does it by a very simple way. We give simple equations with a small set of parameters to calculate the basic geometry of meanders. It seems to us that they can be very useful also for prediction channels and islands during flood of the rivers. Besides, many of peculiarities of real valleys can be included in the model to describe each special case. (For instance, angles α and β may be functions of coordinates). In the present work we demonstrated the common features of the model. Anyone can use it with other parameters and conditions and make his own calculations. The computer code is available (vzavod@mail.ru). We believe that our model will be useful for the water resources managers in the tasks of stream control.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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