MOMENT-ANGLE SPACES OF NONSIMPLE POLYTOPES

A.A.Ayzenberg[1], *V.M.Buchstaber*[2]
[1]Department of Mechanics and Mathematics, Moscow State University, Moscow, Russia
[2]V.A. Steklov Mathematical Institute, RAS, Moscow, Russia

Let P be an n-dimensional polytope with m facets. The moment-angle space $\mathcal{Z}_P$ is defined as an intersection of real quadrics in $\mathbb{C}^m$, which are determined by the embedding of P into $\mathbb{R}^m_\geqslant$. As in the case of simple polytopes, $\dim \mathcal{Z}_P = m + n$ and the space $\mathcal{Z}_P$ carries the canonical action of torus T^m with P as the orbit space. Let K_P be the nerve of the cover of ∂P by the facets of P. Then $\dim K_P \geqslant n-1$ and $\dim K_P = n-1$ whenever P is simple.

1) The space $\mathcal{Z}_P$ is homotopy equivalent to the polyhedral product $(D^2, S^1)^{K_P}$ (see [1]). Note that $\dim(D^2, S^1)^{K_P} > m + n$ for nonsimple polytopes P. This result allows to compute cohomology of $\mathcal{Z}_P$ using the results of [2] which were developed for simple polytopes.

2) The simplicial complex K_P, which was introduced to study $\mathcal{Z}_P$, is an object of an independent interest. We define new combinatorial invariants of polytopes in terms of this complex. An important question is: what are the properties characterizing simplicial complex K_P? This question led us to the notion of nerve-complex — simplicial complex with links of all simplices contractible or homotopically equivalent to a sphere.

References

1. A.A.Ayzenberg, V.M.Buchstaber, Moment-angle spaces and nerve-complexes of convex polytopes // Proceedings of the Steklov Institute of Mathematics, V.275, 2011.

2. V. M. Buchstaber and T. E. Panov, Torus Actions and Their Applications in Topology and Combinatorics // University Lecture, vol. 24, Amer. Math. Soc., Providence, R.I., 2002.